Higher-dim'l pants:

$$\Pi_n := \mathbb{CP}^n \setminus (n+2) \text{ hyperplanes}$$

$$= (\mathbb{C}^*)^n \setminus \{z_1 + \dots + z_n = -1\}$$

Mirror

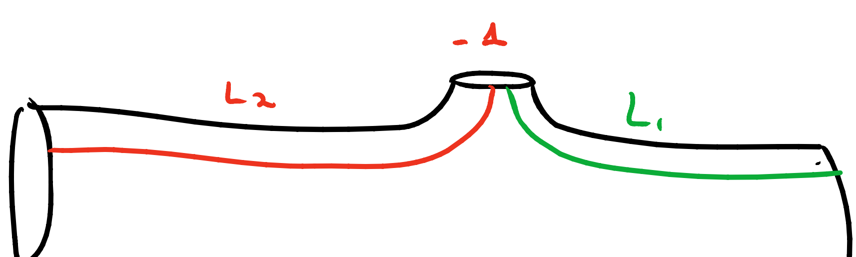
$$\{x_1 \cdots x_{n+1} = 0\} \subset \mathbb{C}^{n+1}$$

OR

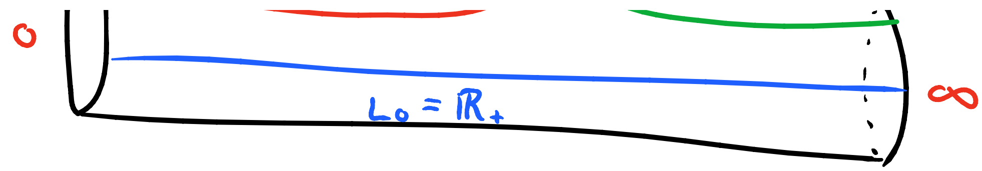
$$(\mathbb{C}^{n+2}, \; W = -x_1 \cdots x_{n+2})$$

$W(\Pi_n)$ (not done carefully yet, but cf Nadler '16)

Π :



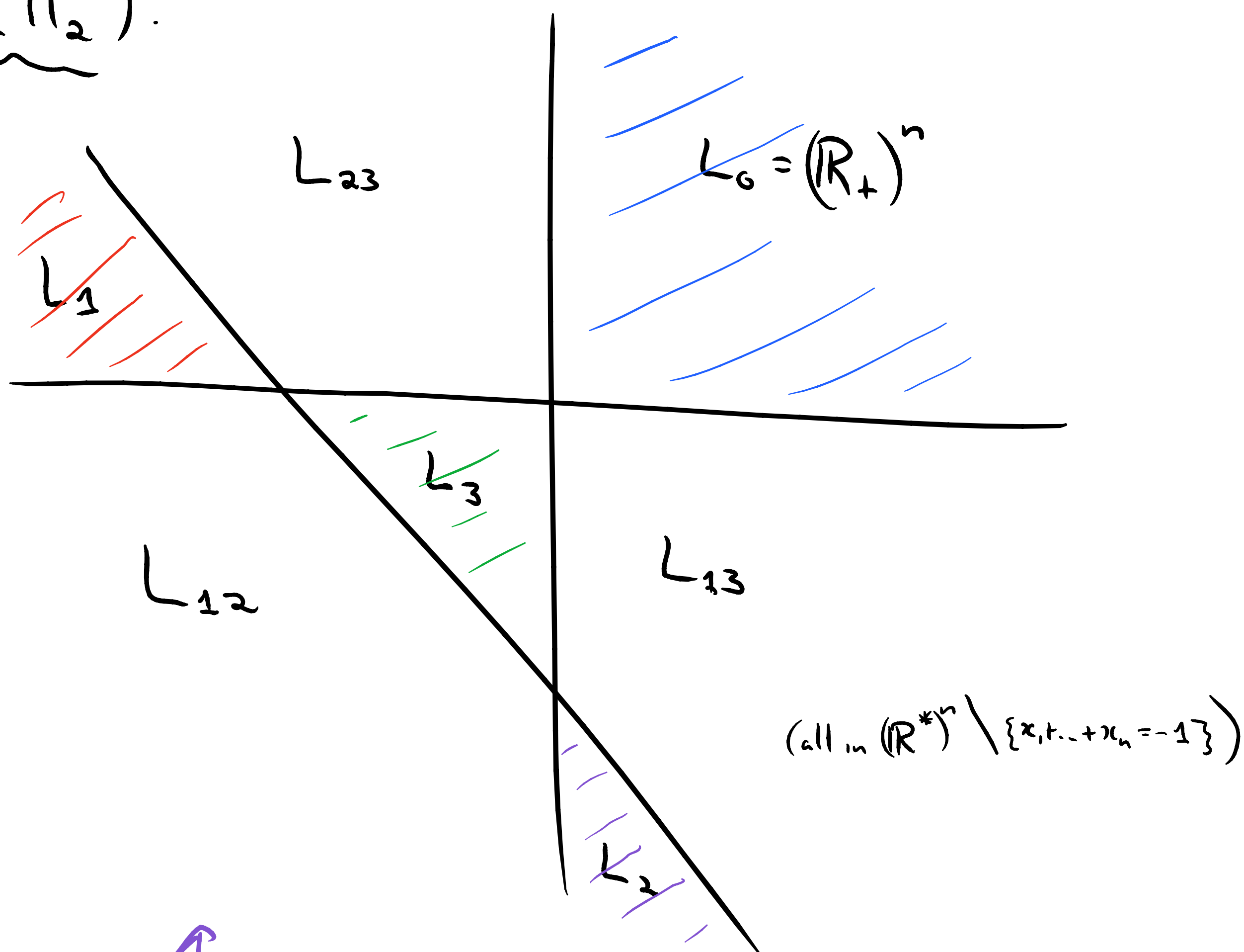
Π_1 :



$$\updownarrow$$

$$\mathcal{O}_{\{x_1x_2=0\}},\quad \mathcal{O}_{\{x_1=0\}},\quad \mathcal{O}_{\{x_2=0\}}$$

$W(\Pi_2)$:



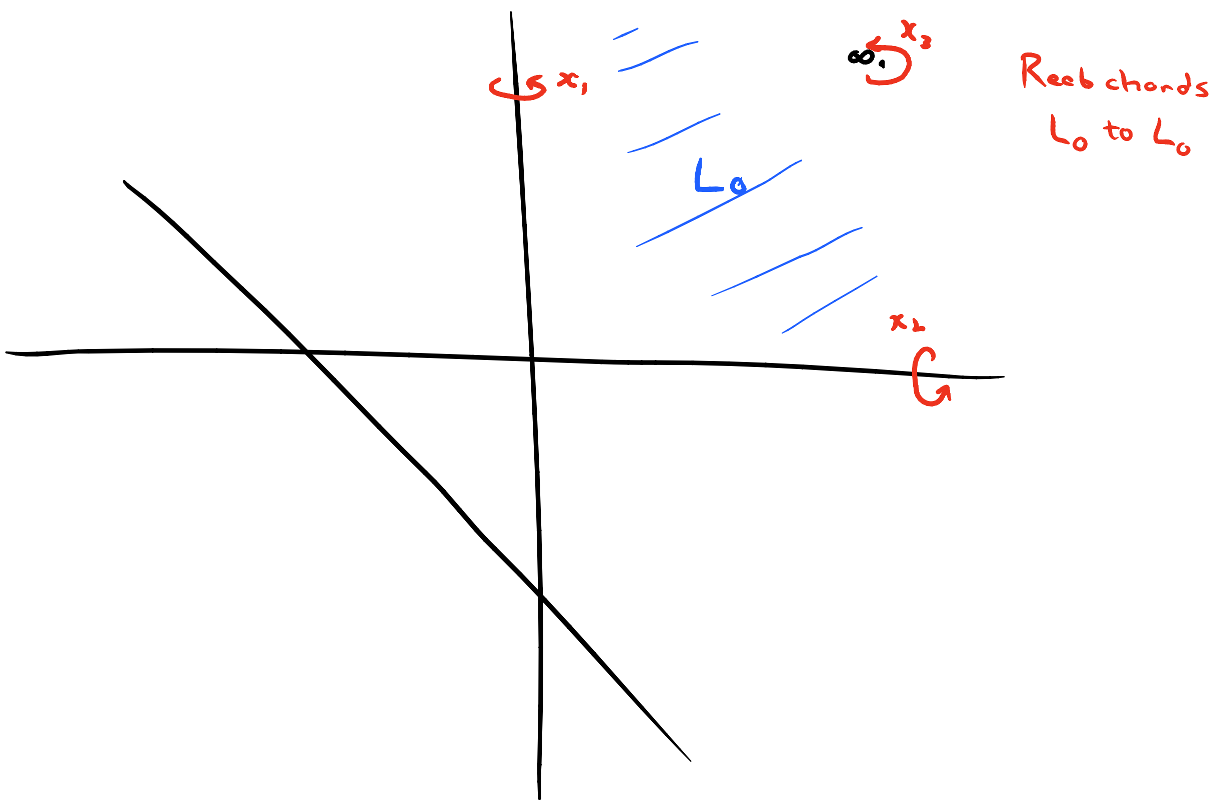
$$X^\vee = \{x_1x_2x_3 = 0\} \subset \mathbb{C}^3$$

$$L_0 \longleftrightarrow \mathcal{O}_{X^\vee}$$

$$L_i \longleftrightarrow \mathcal{O}_{\{x_i=0\}}$$

$$L_{ij} \longleftrightarrow \mathcal{O}_{\{x_i x_j = 0\}}$$

$$CW^*(L_0, L_0) \cong \mathbb{C}[x_1, \dots, x_{n+1}] \big/ (x_1 \cdots x_{n+1}) =: R$$



$$CW^*(L_0, L_i) = \mathbb{C}[x_j, j \neq i]$$
$$= R/x_i$$

... get product structure from this.

Exact triangles

$$L_I \longrightarrow L_J \longrightarrow L_{I\cup J} \longrightarrow$$

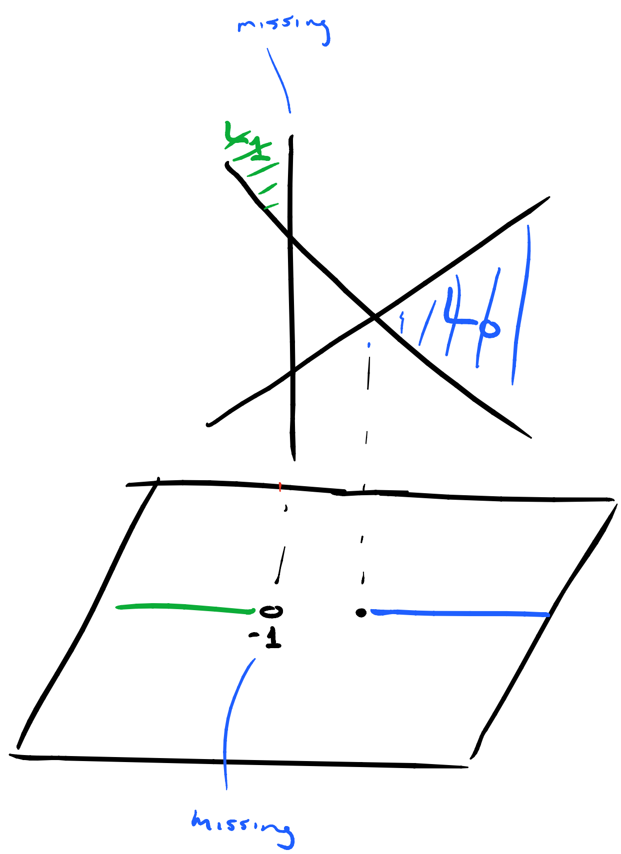
when $I \cap J = \emptyset$

Each L_I fits in a lot of these...

... should recover full A_∞ structure

(not done yet)

$$\begin{array}{c} \Pi_n = (\mathbb{C}^*)^n \setminus \Pi_{n-1} \\ \Big\downarrow {\scriptstyle \Sigma z_i} \\ \mathbb{C} \setminus \{-1\} \end{array}$$



Like Lefschetz fibration
with missing sets

Lagns obtained like thimbles

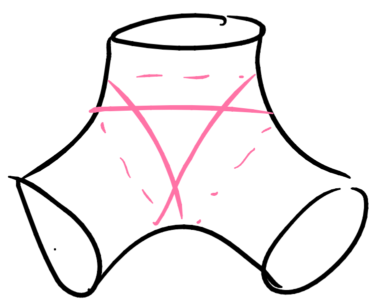
"∂ restriction functor" to missing fiber

$$\begin{tikzcd} \mathcal{W}(\Pi_n) \arrow[r, "/\langle L_0 \rangle"'] \arrow[d, leftrightarrow] & \mathcal{W}(\Pi_{n-1}) \arrow[d, leftrightarrow] \\ D^b\mathrm{Coh}\left(\{x_1 \cdots x_{n+1} = 0\} \subset \mathbb{C}^{n+1}\right) \arrow[r, "/\mathrm{Perf}"] & D^b_{\mathrm{sing}}\left(\mathbb{C}^{n+1}, x_1 \cdots x_{n+1}\right) \end{tikzcd}$$

<u>Compact Lagrangian:</u>

Sheridan's immersed $S^n \looparrowright \Pi_n$

generalizes



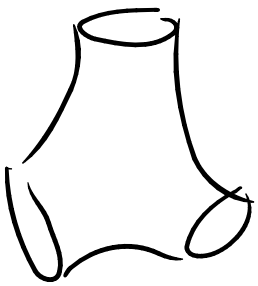
Computed HF

+ deformations upon "orbifold compactifications"

$\rightsquigarrow$ Cover $\deg \leq n+2$

$$\rightsquigarrow \mathcal{F}\left(\begin{array}{c}\text{Fano or CY}\\ \text{hypersurface}\\ \text{in } \mathbb{CP}^{n+1}\end{array}\right) \simeq D^b(\underbrace{\text{Mirror}})$$

$$\left(\mathbb{C}^{n+2}, W = \begin{array}{l} -z_1\cdots z_{n+2} \\ \quad + \sum z_i^k \\ \qquad + \dots \end{array}\right)$$



$$D^b Coh(\Pi_n) \xrightarrow{\simeq} \mathcal{W}(\underbrace{\mathbb{C}^{n+2}}_{= Y}, \underbrace{-z_1\cdots z_{n+2}}_{= W})$$

fibrewise wrapped

[Abouzaid - A]

(for more general constructions, see [Abouzaid - Ganatra] and [Sylvan])

(1) Can define in this setting "fibrewise wrapped"

$$\mathcal{W}(Y, W)$$

(2) Can construct object L_0, conj. generates

(3) Calculate $\mathrm{End}(L_0) \cong \mathbb{k}[x_1^{\pm 1}, \dots, x_{n+3}^{\pm 1}] \big/ (\sum x_i + 1 = 0)$

Rmk: Sylvan's set-up

Liouville mfld

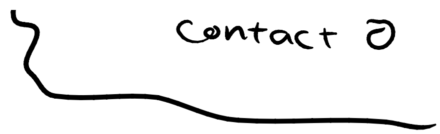
+ "stops" on

contact ∂

General Setup

(see P. Clarke)

General Setup



[Abouzaid - A - Katzarkov]

$\rho : A \to \mathbb{R}$

$$H^{n-1} = \left\{ f_t = \sum_{\alpha \in A \subseteq \mathbb{Z}^n} c_\alpha t^{\rho(\alpha)} x^\alpha = 0 \right\} \subset (\mathbb{C}^*)^n$$

$$|t| \ll 1$$

Construct "generalized SYZ mirror" to H

(Y, W) where Y = toric CY $(n+1)$-fold

moment polytope

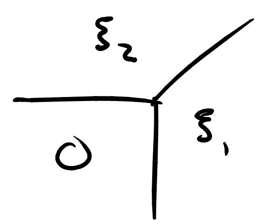
$$\Delta_Y = \left\{ (\xi_1, \dots, \xi_n, \eta) \in \mathbb{R}^n \times \mathbb{R} \;\middle|\; \eta \geq \operatorname{Trop} f(\xi_1, \dots \xi_n) \right\}$$

where

$$\operatorname{Trop} f(\xi) = \max_{\alpha \in A} \left(\langle \alpha, \xi \rangle - \rho(\alpha) \right)$$

(Pants: $f = 1 + x_1 + \dots + x_n$

$\operatorname{Trop} = \max(0, \xi_1, \dots, \xi_n)$



)

$$W = -z^{(0,0,\dots,0,1)}$$

$W = -z$

toric monomial vanishing to order 1 on each facet

Compactify to $\overline{H} \subset V$ toric Fano

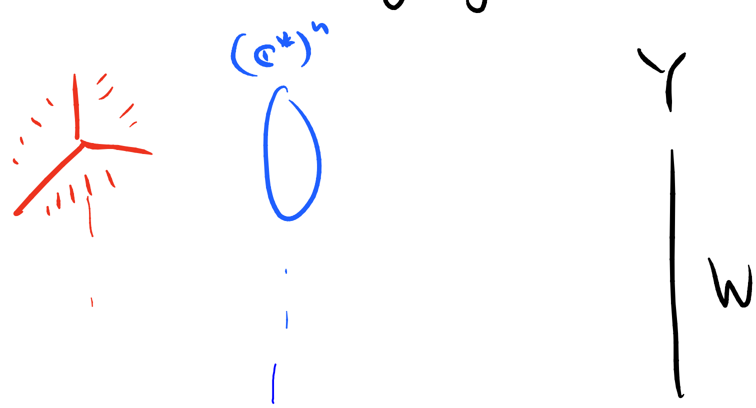
$\rightsquigarrow$ W acquires extra terms, one for each facet V

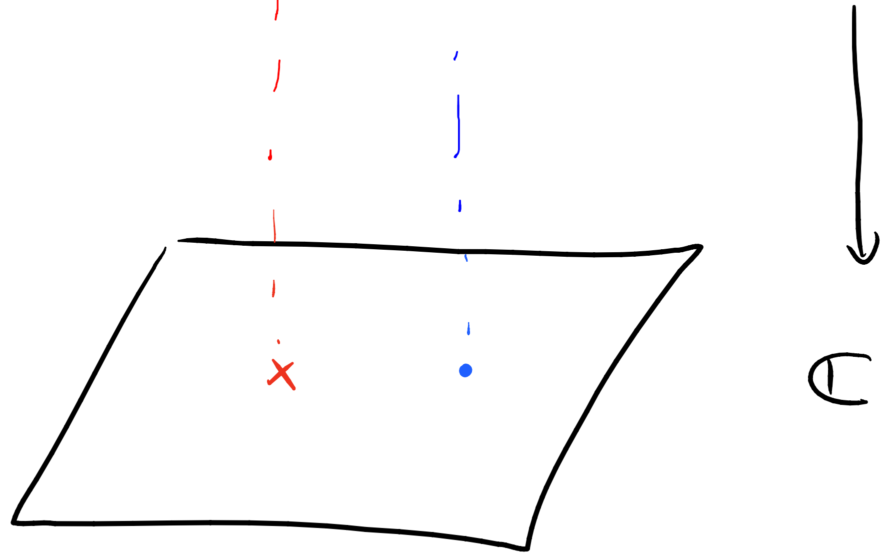
[Abouzaid–A.]

For mirrors of $H \subset (\mathbb{C}^*)^n$, can still define $W(Y, w)$ and get (conjecturally) generating L_0 object with

$$\mathrm{End}(L_0) \cong \Bbbk[x_1^{\pm 1}, \dots, x_n^{\pm 1}]/(f_t) \cong \mathrm{End}(\mathcal{O}_H)$$

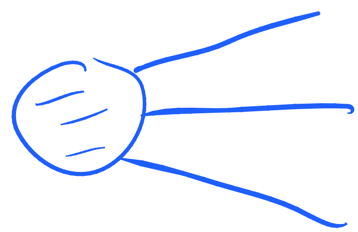
(1) Fibrewise-wrapped category?





Objects: Properly embedded Lagns $L \subset Y$

s.t. $W(L) \overset{\text{(looks like)}}{=}$



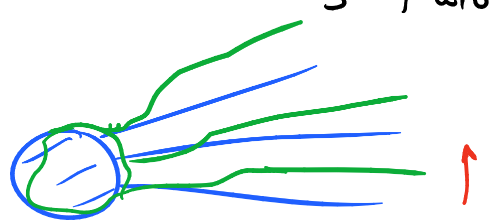
union of rays asymptotic to $(e^{i\theta}\mathbb{R}_+)$ straight radial lines, with $\theta \neq \pi$

Let ρ_t = flow in $\mathbb{C}$:

id in disc

takes straight $\to$ straight

bending $\uparrow$ w/o crossing π



Lift via symplectic parallel transport

$\rho_t(L)$ Lagrangian

ϕ^t = flow of Ham $H : Y \to \mathbb{R}$

where H is...

invt under $\|$ transport, fibrewise proper,

"linear at ∞"

Combine: $L^t := \varphi^t_{\rho^t} L$

- Lagns we want:

  fibrewise, as $|z_i| \to \infty$, $\arg(z_i) \longrightarrow$ constant

- "Separation at ∞": For $t \notin \mathbb{Z}$, these constants are different

  i.e. $\Rightarrow$ Say $L \pitchfork_{\infty} L'$ if these constants are different and also for L^t, L' with $t \notin \mathbb{Z}$

- For $t > 0$ small enough, $L^t \sim$ graph of d(proper function) over L

Define: $\hom(L, L') = \varinjlim_{t \to +\infty} CF(L^t, L')$

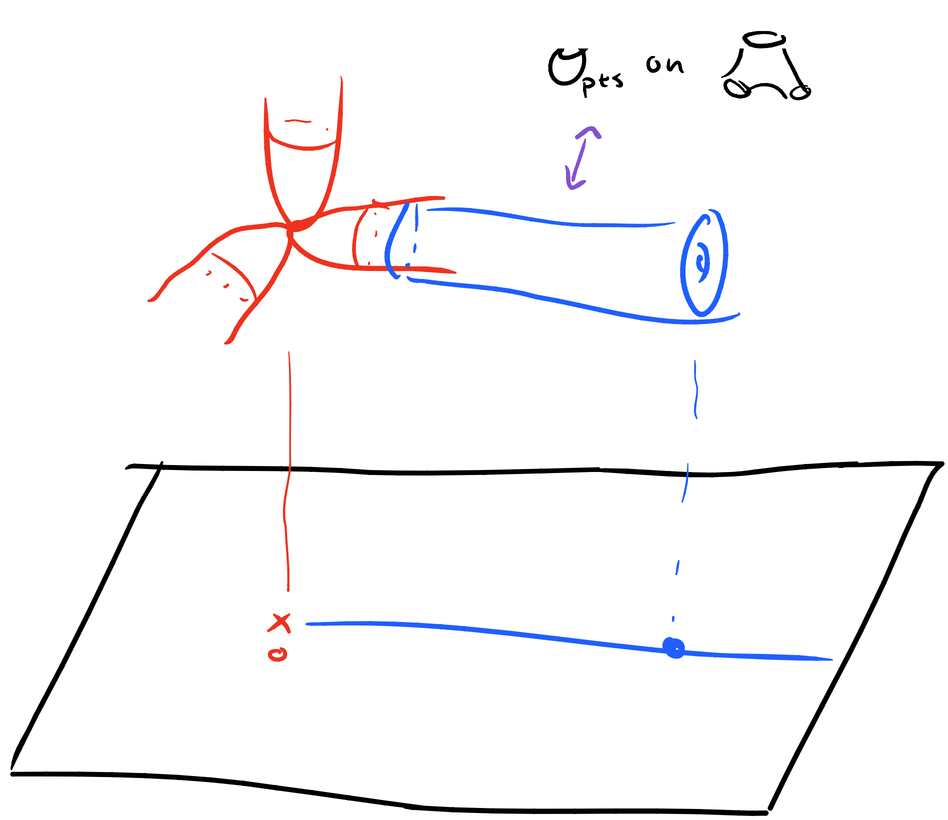
w/ continuation = mult^n by "id: $t \to t + \varepsilon$"

(2) L :

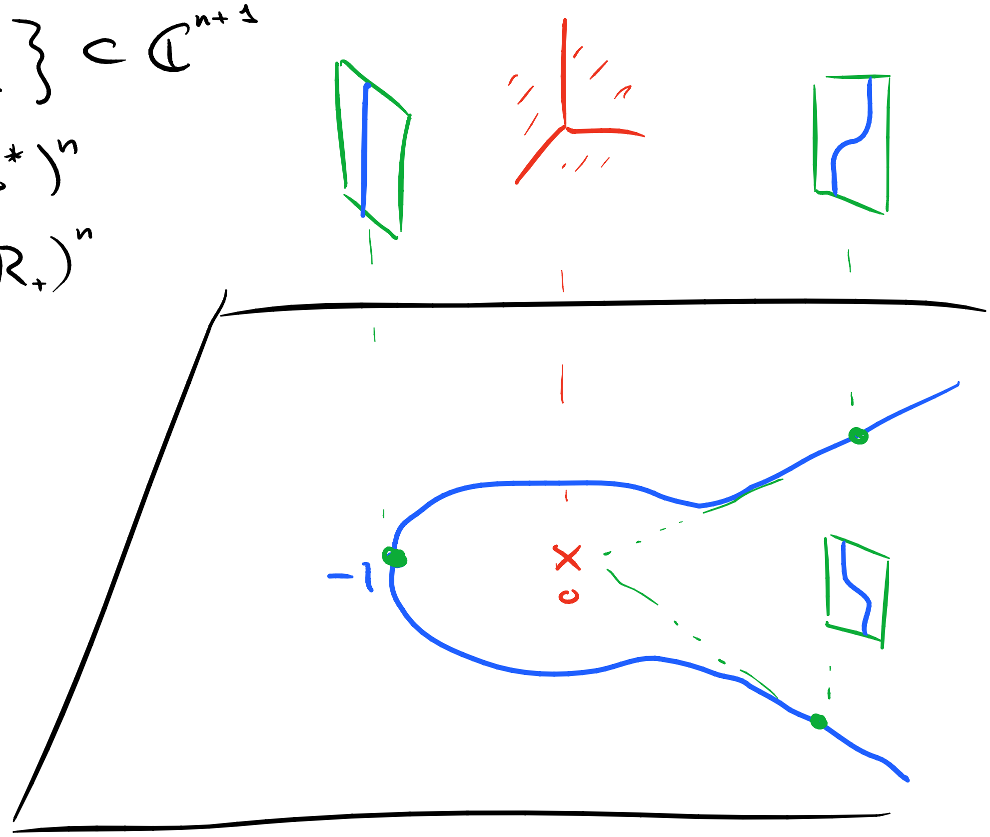
$\mathcal{O}$ on

(2) L_0 :

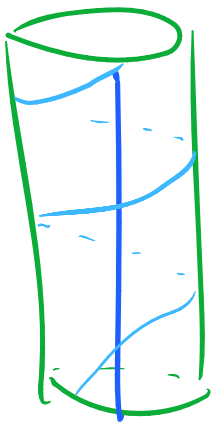
$$\mathbb{C}^3 \longrightarrow \mathbb{C}$$



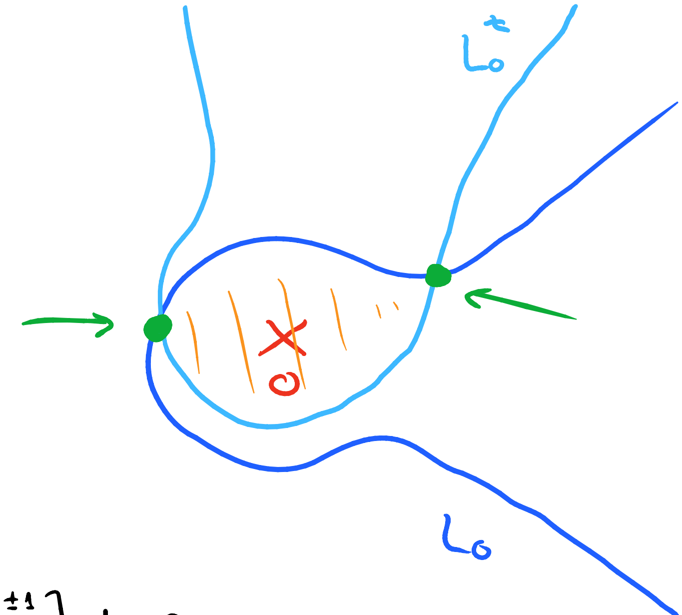
$$\{\textstyle\prod z_i = 1\} \subset \mathbb{C}^{n+1}$$
$$\simeq (\mathbb{C}^*)^n$$
$$\supset (\mathbb{R}_+)^n$$

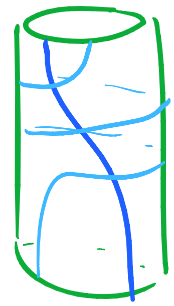


(3) Calculation of $\hom(L_0, L_0)$



$(\mathbb{C}^*)^n$ $\Bbbk[x_i^{\pm 1}]$ deg 0





$\Bbbk[x_i^{\pm 1}]\cdot\Theta$
deg -1

$\mu^{k\geq 3}$ vanish

Differential: $\Theta \mapsto f(x_i)$
$= 1 + \sum x_i$ for p.o.p.

$$HF^{-1} = 0$$

$$HF^{0} = \Bbbk[x_i^{\pm 1}]/(f)$$