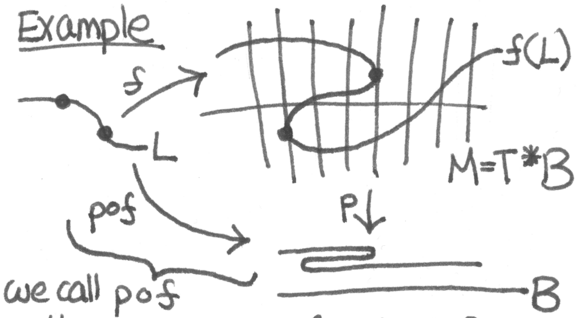
THE SIMPLIFICATION OF SINGULARITIES OF LAGRANGIAN & LEGENDRIAN FRONTS

Let $\mathcal{F}$ be a foliation of a symplectic or contact manifold M by Lagrangian or Legendrian leaves.

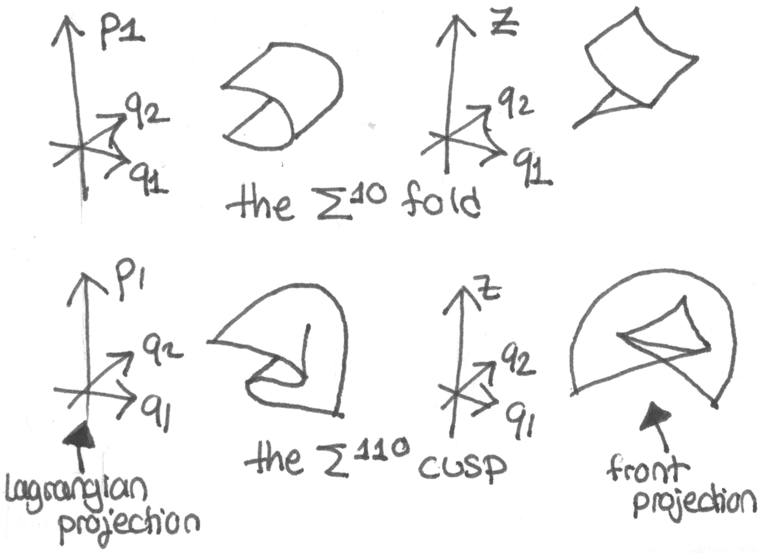
Def. A singularity of tangency of a Lagrangian or Legendrian embedding $f: L \to M$ is a point $q \in L$ such that $df(T_qL) \cap T_{f(q)}\mathcal{F} \neq 0$.

Example



we call $p \circ f$ the Lagrangian front of f

The simplest singularities

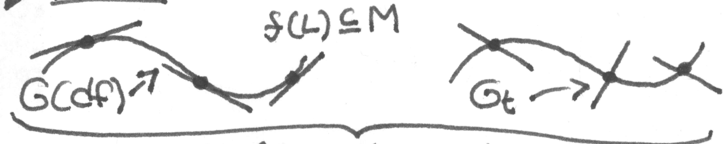


These singularities are also known by the name of caustics. Generically, caustics are terrible and cannot be explicitly understood. Moreover, there exists a homotopy theoretic obstruction to the simplification of singularities

Def. A tangential rotation of a Lagrangian or Legendrian embedding $f: L \to M$ is a deformation $G_t: L \to \Lambda_n(M)$, $t \in [0,1]$, of the Gauss map $G_0 = G(df)$ such that G_t covers $f = \pi \circ G_t$.

Remark: $\Lambda_n(M) \xrightarrow{\pi} M$ denotes the Lagrangian or Legendrian Grassmannian of M and $G(df)(q) = df(T_qL)$ is the Gauss map of f.

Picture:



The tangential information is decoupled from the underlying embedding.

We prove the following h-principle.

Thm (AG, '16) Suppose that there exists a tangential rotation G_t of f such that $G_1 \pitchfork \mathcal{F}$. Then there exists an ambient Hamiltonian isotopy $\varphi_t: M \to M$ such that the caustics of $\varphi_1 \circ f$ consist only of Σ^{10} folds.

'Proof': The key ingredients are a refinement of Eliashberg & Mishachev's holonomic approximation lemma (inspired by Gromov's iterated convex hull extensions) and a local wrinkling model for Lagrangians & Legendrians based on Eliashberg & Mishachev's wrinkled embeddings theorem.

APPLICATIONS

Ekholm's Morse flow-trees only work when the caustics consist only of Σ^{10} folds. In family Floer cohomology of Fukaya and Abouzaid, Σ^{10} folds correspond to chain-level generators appearing & disappearing in a Morse birth/death bifurcation. We expect many more...

Sketch: